Math 760

# Chapter 2 HW

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## 4. When and exist, prove each of the following.

*Hint*: Part (a) can be proved by noting that , I = I’, and . Part (b) follows from

### (a)

We know:

I = I’(

Therefore, and . And because of this, this means is the inverse of A’,

### (b)

We know:

If a matrix B s.t. BA = AB = I, then B is called the inverse of A and is denoted by . And,

So following all that, AB has inverse

## 5. Check that

**is an orthogonal matrix.**

We want to know if , to prove that Q is an orthogonal matrix. The is:

## [,1] [,2]  
## [1,] 0.3846154 -0.9230769  
## [2,] 0.9230769 0.3846154

Now, we’ll multiply the two matrices and see if the answer is an identity matrix.

## [,1] [,2]  
## [1,] 1 0  
## [2,] 0 1

Thus, Q is an orthogonal matrix.

## 6. Let

### 

### (a) Is A symmetric?

## [1] TRUE

## 9 -2 -2 6 = 9 -2 -2 6

Yes, **A** is symmetric.

### 

### (b) Show that A is a positive definite

## [1] TRUE

## The eigenvalues of A are: 10 5 .

Because the eigenvalues are positive, A is a positive definite

## 

## 14. Show that , and have the same eigenvalues if Q is orthogonal.

*Hint: Let be an eigenvalue of* ***A****. Then . By Exercise 2.13 and Result 2A.11(e), we can write , since Q’Q = I.*

With the hint in mind, we can write: . If Q is orthogonal, then is also an eigenvalue of QAQ’.

## 20. Determine the square-root matrix , using the matrix A in Exercise 2.3. Also, determine , and show that .

is

## [,1] [,2]  
## [1,] 1.3763819 0.3249197  
## [2,] 0.3249197 1.7013016

is

## [,1] [,2]  
## [1,] 0.7608452 -0.1453085  
## [2,] -0.1453085 0.6155367

Now, we prove .

## [,1] [,2]  
## [1,] 1 0  
## [2,] 0 1

## [,1] [,2]  
## [1,] 1 0  
## [2,] 0 1

## 

## 26. Use as given in Exercise 2.25

## [,1] [,2] [,3]  
## [1,] 25 -2 4  
## [2,] -2 4 1  
## [3,] 4 1 9

### 

### (a) Find .

=

## [1] 0.2666667

### 

### (b) Find the correlation between and .

We want to know:

We know:

Then:

, where

, where

From there, we can find the variances and covariance.

## [1] 25

Because we have the X’s multiplied by a constant for , the variance will be in this form: (2-43).

## [1] 3.75

The same applies for the covariance:

=

## [1] 1

Finally, we can plug everything in.

## [1] 0.1032796

## 

## 32. You are given the random vector with mean vector , and variance-covariance matrix

**Partition X as**

**Let**

**and**

**and consider the linear combinations and . Find**

### 

### (a)

## [,1]  
## [1,] 2  
## [2,] 4

### 

### (b)

## [,1]  
## [1,] -2  
## [2,] 6

### 

### (c)

## [,1] [,2]  
## [1,] 4 -1  
## [2,] -1 3

### (d)

## [,1] [,2]  
## [1,] 9 1  
## [2,] 1 5

### 

### (e)

## [,1]  
## [1,] -1  
## [2,] 3  
## [3,] 0

### 

### (f)

## [,1]  
## [1,] 2  
## [2,] 2

### 

### (g)

## [,1] [,2] [,3]  
## [1,] 6 1 -1  
## [2,] 1 4 0  
## [3,] -1 0 2

### 

### (h)

## [,1] [,2]  
## [1,] 12 9  
## [2,] 9 24

### 

### (i)

## [,1] [,2] [,3]  
## [1,] 0.5 -0.5 0  
## [2,] 1.0 -1.0 0

### 

### (j)

## [,1] [,2]  
## [1,] 0 0  
## [2,] 0 0

## 34. Consider the vectors and . Verify the Cauchy-Schwarz inequality .

Let’s find the right side of the inequality first.

(b’b) is:

## [,1]  
## [1,] 21

(d’d) is:

## [,1]  
## [1,] 15

The product of (b’b)(d’d) is:

## [,1]  
## [1,] 315

Now, the left side:

## [,1]  
## [1,] 169

Finally, let’s plug it all in.

The Cauchy-Schwarz inequality holds!

## 

## 42. Repeat Exercise 2.41, but with

**Let**

### 

### (a) Find E(AX), the mean of AX.

We know: . Therefore, the mean of **AX** is:

## [,1]  
## [1,] 1  
## [2,] 9  
## [3,] 3

### 

### (b) Find Cov(AX), the variances and covariances of AX.

We know: . Therefore the variances and covariances of

**AX** is:

## [,1] [,2] [,3]  
## [1,] 4 0 0  
## [2,] 0 12 0  
## [3,] 0 0 24

### 

### (c) Which pairs of linear combinations have zero covariances?

All pairs of linear combos have zero covariances as shown above. This is because the covariance matrix is structured as such:

**Code**

knitr::opts\_chunk$set(echo = FALSE)  
library(expm)  
library(Matrix)  
library(matrixcalc)  
Qmat <- c(5/13,12/13,-12/13,5/13)  
Q <- matrix(Qmat, nrow = 2, ncol = 2, byrow = TRUE)  
QT <- t(Q)  
print(QT)  
Q%\*%QT  
aMat <- c(9,-2,  
 -2,6)  
A <- matrix(aMat, nrow = 2, ncol = 2, byrow = TRUE)  
isSymmetric(A)  
AT <- t(A)  
cat(A," = ", AT)  
is.positive.definite(A)  
ev <- eigen(A)  
value <- ev$values  
cat("The eigenvalues of A are:", value, ".")  
aMat <- c(2,1,1,3)  
A <- matrix(aMat, nrow = 2, ncol = 2, byrow = TRUE)  
sqrtMat <- sqrtm(A)  
sqrtMat  
neg <- solve(sqrtMat)  
neg  
AAneg <- sqrtMat%\*%neg  
negAA <- neg%\*%sqrtMat  
AAneg  
negAA  
sigMat <- c(25, -2, 4,  
 -2, 4, 1,  
 4, 1, 9)  
sigma <- matrix(sigMat, nrow = 3, ncol = 3, byrow = TRUE)  
sigma  
p13 <- 4/(sqrt(25)\*sqrt(9))  
p13  
var1 <- sigma[1,1]  
var1  
var2 <- 0.25\*(sigma[2,2] + 2\*sigma[2,3] + sigma[3,3])  
var2  
cov <- 0.5\*(sigma[1,2] + sigma[1,3])  
cov  
rho <- cov/(sqrt(var1)\*sqrt(var2))  
rho  
# A  
aMat <- c(1,-1,1,1)  
A <- matrix(aMat, nrow = 2, ncol = 2, byrow = TRUE)  
# B  
bMat <- c(1,1,1,  
 1,1,-2)  
B <- matrix(bMat, nrow = 2, ncol = 3, byrow = TRUE)  
  
# mu  
mu1Mat <- c(2,4)  
mu1 <- matrix(mu1Mat, nrow = 1, ncol = 2, byrow = TRUE)  
mu2Mat <- c(-1,3,0)  
mu2 <- matrix(mu2Mat, nrow = 1, ncol = 3, byrow = TRUE)  
  
# sigma  
sigMat <- c(4,-1,0.5,-0.5,0,  
 -1,3,1,-1,0,  
 0.5,1,6,1,-1,  
 -0.5,-1,1,4,0,  
 0,0,-1,0,2)  
sigma <- matrix(sigMat, nrow = 5, ncol = 5, byrow = TRUE)  
t(mu1)  
A %\*% t(mu1)  
sig11 <- sigma[1:2,1:2]  
sig11  
A %\*% sig11 %\*% t(A)  
t(mu2)  
B %\*% t(mu2)  
sig22 <- sigma[3:5,3:5]  
sig22  
B %\*% sig22 %\*% t(B)  
sig12 <- sigma[3:5,1:2]  
t(sig12)  
A %\*% t(sig12) %\*% t(B)  
bMat <- c(2,-1,4,0)  
dMat <- c(-1,3,-2,1)  
  
# matrix  
b <- matrix(bMat, nrow = 1, ncol = 4, byrow = TRUE)  
d <- matrix(dMat, nrow = 1, ncol = 4, byrow = TRUE)  
b2 <- b %\*% t(b)  
b2  
d2 <- d %\*% t(d)  
d2  
bd2 <- b2 %\*% d2  
bd2  
bd <- b %\*% t(d)  
bd^2  
sigMat <- c(3,1,1,1,  
 1,3,1,1,  
 1,1,3,1,  
 1,1,1,3)  
aMat <- c(1,-1,0,0,  
 1,1,-2,0,  
 1,1,1,-3)  
muMat <- c(3,2,-2,0)  
  
# matrix  
sigma <- matrix(sigMat, nrow = 4, ncol = 4, byrow = TRUE)  
A <- matrix(aMat, nrow = 3, ncol = 4, byrow = TRUE)  
mu <- matrix(muMat, nrow = 1, ncol = 4, byrow = TRUE)  
A %\*% t(mu)  
A %\*% sigma %\*% t(A)